

Quark cluster signatures in deuteron electromagnetic interactions

C.E. Carlson

*Physics Department, College of William and Mary
Williamsburg, VA 23187, USA*

K.E. Lassila

*Department of Physics and Astronomy
Iowa State University, Ames, IA 50011, USA*

Abstract. A suggestion is made for distinguishing 2N and 6q short range correlations within the deuteron. The suggestion depends upon observing high momentum backward nucleons emerging from inelastic electromagnetic scattering from a deuteron target. A simple model is worked out to see the size of effects that may be expected.

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1 Introduction

In deep inelastic scattering upon deuterons or heavier nuclei, nucleons or other hadrons can emerge backward to the direction defined by the incoming photon in the target rest frame. The backward nucleons do not, we believe, come from the nucleon or the quark that was struck [1]. Rather, the backward nucleons come from the debris that remains after the item that was struck is driven strongly forward. We have commented [2, 3] upon using neutrino production of backward protons [4, 5, 6, 7] to explore short distance quark configurations and here wish to use deep inelastic electromagnetic interactions to similar ends.

We distinguish two extreme cases. In a 2-nucleon or 2N correlation the nucleons maintain their characters as neutrons and protons no matter what their separation is. To get a backward proton from the deuteron, one must strike the neutron. This will break up the bound state, and the proton will emerge with the Fermi momentum it has at the moment of breakup, and the Fermi momentum will be backwards in the case of interest.

The other extreme case is the 6-quark or 6q cluster. Here we mean a “kneaded” 6q, $uuuddd$, object with all the quarks in relative S-states. The flavor-spin-color wave function is unique (for overall deuteron quantum numbers) and is not equivalent to two nucleons lying at the same spatial point. Emitting a backward proton begins with one quark being struck and driven forward. The proton must be formed out of the remaining 5 quarks, plus possible higher Fock components, and the process of forming hadrons we refer to as the “fragmentation” of the 5-quark residuum. The term fragmentation follows common usage for the production of hadrons from any color non-singlet QCD object, quark and gluon jets being the most familiar. In the breakup of this 5-quark residuum, we persist with the nomenclature “fragmentation” although recombination may be the

process chiefly at work. In any case, the 6q model can produce a backward proton spectrum which agrees with data from neutrino reactions, for backward hemisphere proton momentum above about 300 MeV [2]. However, so can the 2N model with enhanced high momentum components [8]. We need a more detailed indicator to test the two models. For weak interactions, ratios of neutrino and antineutrino induced backward proton production cross sections cancel much of what is unknown in either the 2N or 6q model and yet give predicted results which are not the same for the two models [3]. Similar opportunities should also exist for electromagnetic interactions.

Here we suggest a characteristic in the spectrum of backward nucleons which is reliably different for the two scenarios of the short range configuration, and which allows electromagnetic experiments to adjudicate between them. A suggestion for a ratio to examine at fixed backward proton momentum and varying Björkén x is given in section 2, along with some numerical estimates of the differences between the two models. Further comments on possibilities at CEBAF, Fermilab, or elsewhere are made in section 3. Incidentally, an observation regarding changes in average Björkén x with varying backward nucleon momentum, which was originally made in the context of the 2N model, should be reasonably true for most any model and we also comment on this in section 3. We conclude in section 4.

2 Signatures of short distance correlations

2.1 *General*

The cross section for inelastic electromagnetic scattering of a lepton from a stationary target is (in the scaling region and neglecting σ_L/σ_T)

$$\frac{d\sigma}{dx dy} = \frac{4\pi \alpha_{em}^2 m_N E}{Q^4} \left(1 + (1 - y)^2\right) F_2(x, Q^2) \quad (1)$$

where E and E' are the incoming and outgoing lepton energies, ν is the difference between them, $y = \nu/E$, $x = Q^2/2m_N\nu$, and F_2 (whose Q^2 dependence will generally be tacit) is

$$F_2(x) = \sum e_i^2 x q_i(x) \quad (2)$$

where e_i is the quark charge in units of proton charge and q_i is the distribution function for a quark of flavor i .

2.2 Backward nucleons from $2N$ correlations

We will speak of observing a backwards proton for the sake of definiteness; observing a backwards neutron is essentially similar.

Some things change in the above formula when a backwards proton is observed. The neutron, which is the struck particle, is not at rest in the lab frame. Then the momentum fraction of the struck quark relative to the neutron is not x but rather ξ ,

$$\xi = \frac{x}{2 - \alpha}, \quad (3)$$

where $\alpha = (E_p + p^z)/m_N$ is the light cone momentum fraction of the backward proton with p^z positive for a backward proton. One should also replace $m_N E$ by its corresponding Lorentz invariant,

$$m_N E \rightarrow p_n \cdot k = m_N E (2 - \alpha). \quad (4)$$

Also, if we want a backward proton, there is a further factor of the probability density for finding the proton with its observed momentum at the moment the neutron was struck. Thus

$$\sigma_{2N} \equiv \frac{d\sigma_{2N}}{dx dy d\alpha d^2 p_T} = \frac{4\pi \alpha_{em}^2 m_N E}{Q^4} (1 + (1 - y)^2) (2 - \alpha) F_{2n}(\xi) \cdot |\psi(\alpha, p_T)|^2, \quad (5)$$

where p_T is the transverse momentum of the backward proton and we used the light cone wave function normalized by

$$\int d\alpha d^2 p_T |\psi(\alpha, p_T)|^2 = 1. \quad (6)$$

The neutron structure function F_{2n} is (we shall suppose) known. The test we propose is to measure the cross section for backward nucleon production at a variety of x and α and examine the ratio

$$R_1 = \frac{\sigma_{meas}/K}{F_{2n}(\xi)}. \quad (7)$$

Here, σ_{meas} is the measured differential cross section and K is the factor

$$K = \frac{4\pi \alpha_{em}^2 m_N E}{Q^4} (1 + (1 - y)^2). \quad (8)$$

The signature of a two nucleon correlation model is that this ratio is independent of x for any fixed α and p_T . Of course, how useful this signature is depends upon how different we may expect the result to be for a 6q cluster. This we shall see in the next section.

2.3 Backward nucleons from 6q clusters

For the case of electromagnetic scattering from the 6q state, we have basically the convolution of $F_2^{(6)}$ with the fragmentation functions of the five (or more, in general) quark residuum. Since the quarks in the residuum depend on which flavor quark was struck, we must write

$$\sigma_{6q} \equiv \frac{d\sigma_{6q}}{dx dy d\alpha d^2 p_T} = K \sum_i x e_i^2 V_i^{(6)}(x) \cdot \frac{1}{2-x} D_{p/5q}(z, p_T). \quad (9)$$

Here $V_i^{(6)}$ is the distribution function for a valence quark in a six quark cluster, the sum is over quark flavors, and $D_{p/5q}$ is the fragmentation function for the 5q residuum, i.e., the probability density per unit z and p_T for finding a proton coming from the 5q cluster. It is tacit that the correct 5q cluster, either $u^2 d^3$ or $u^3 d^2$, is chosen. Argument z is the fraction of the residuum's light-cone longitudinal momentum that goes into the proton,

$$z = \frac{\alpha}{2-x}; \quad (10)$$

the factor $(2-x)^{-1}$ comes because $D_{p/5q}$ is probability per unit z in its definition and we quote the differential cross section per unit α . The formula is written for high backward proton momentum, where we can expect the 5q residuum and hence the actual 6q initial state to dominate.

Neither $F_2^{(6)}$ nor $D_{p/5q}$ can be said to be known. However, since a large fraction of the short range part of the baryon number two state may be in a 6-quark cluster, we should make the best guess as to what these functions might be and see how large a difference it could make experimentally to have significant 6q cluster contributions, at least in given regions of phase space.

Estimates of $F_2^{(6)}$ in a model where the nuclei are treated as containing some fraction 6q clusters have been given by Lassila and Sukhatme [9]. They chose their quark distributions beginning with quark counting rules and then fine tuned with physical logic to describe the EMC data. For completeness, the three parameterizations they present are recorded in the Appendix.

The fragmentation function is even less well known since there is no complete body of data to check it against. The counting rules suggest a cubic dependence, as (unnormalized)

$$D_{p/q^5}(z) \propto (1-z)^3 \quad (11)$$

for $z \rightarrow 1$ and for zero p_T . We shall use this form, although we keep in mind the possibility that higher order contributions or renormalization group considerations could somewhat alter the power, as they do in many parameterizations of the quark distributions in nucleons.

If the high momentum backward protons come from a 6-quark cluster, then $\sigma_{meas} = \sigma_{6q}$ and the experimental ratio R_1 should be given by

$$R_1 = R_1^{(6)} = \frac{F_2^{(6)}(x) D_{p/5q}(z, p_T)}{(2-x)(2-\alpha) F_{2n}(\xi)}. \quad (12)$$

There is no reason for $R_1^{(6)}$ to be independent of x for fixed α and p_T . We plot this R_1 in Figs. 1 and 2 for $p_T = 0$ and specified α . Some old and simple quark distributions of Carlson and Havens [10] were used to get F_{2n} in Fig. 1 and the CTEQ1L [11] distributions were used in Fig. 2. The difference between what is seen and the horizontal line expected from a pure 2N correlation model is not negligible. That the ratio goes to a finite value as we reach the kinematic limit $x = 2 - \alpha$ in Fig. 1 has to do with the fact that both the 5q fragmentation function and the dominant quarks in the nucleon approach their end points cubically in our models. A dive to zero or a flight to infinity is not precluded in real life, and the latter is seen in Fig. 2.

3 Commentary

3.1 *Potential dominance of 6q cluster*

We expect that a correct description of the deuteron would have a neutron and a proton treated as in the 2N model when they are far apart. As they get closer, QCD processes such as gluon recombination [12] will surely occur and affect first the ocean parton distributions. It is something of a simplification to think of a deuteron as just a combination with a large fraction pure 2N state with a small fraction (perhaps 5%, from wave function overlap estimates [13]) 6q state added in.

However, we emphasize that while a 6q cluster may be a small part of the deuteron overall, it could be a large fraction of the short range part of the deuteron wave function. The deviation from what is expected in a pure 2N correlation could be as large as is shown in our Figures. The situation is not like the EMC effect where the effects of the 6q cluster are diluted by the mostly ordinary collection of nucleons in the target. Here we can select events in a phase space region to enhance 6q cluster effects. The backward proton is a tag that emphasizes the 6q cluster and—if it is there at all—it will dominate

the cross section for large enough backward proton momentum.

It is necessary to have data at fairly high backward momenta. No one doubts that at low relative momentum or long distances the deuteron is a neutron plus a proton. How high is needed? We suggest 300 MeV/c is a good starting point, based on existing backward proton data from deep inelastic neutrino scattering and studies of the backward proton spectrum in that process using 6q cluster ideas.

3.2 *Falling $\langle x \rangle$ with increasing α*

Let us point out a piece of kinematics. From momentum conservation one has $0 \leq x \leq (2 - \alpha)$. Hence, unless the x distribution has a bizarre shape, one expects that $\langle x \rangle_\alpha$ —the average value of x at fixed α —decreases as α increases. This was pointed out in Ref. [1] in the context of the 2N model, and was initially suggested as a test of that model. However, the result should be produced by any model, so finding the trend in the data is not startling.

The two-nucleon correlation model does give a specific result that $\langle x \rangle_\alpha$ falls to zero linearly as $2 - \alpha$. In contrast, the six-quark cluster model may or may not fall quite linearly. It depends on the specific implementation of the model.

The rest of this subsection attempts to show why the two-nucleon result for $\langle x \rangle_\alpha$ is independent of the internal details of that model, but similar manipulations for other models do not lead to such definite results. It has to do with the way the cross section factors.

For the two-nucleon model, the structure of the formula for the cross section differential in x and α is,

$$P(x, \alpha) = \frac{dN}{dx d\alpha} = f(\xi) \cdot \frac{|\psi(\alpha)|^2}{2 - \alpha}, \quad (13)$$

where ξ is defined earlier. An elementary calculation gives

$$\langle x \rangle_\alpha = \frac{\int_0^{2-\alpha} dx x P(x, \alpha)}{\int_0^{2-\alpha} dx P(x, \alpha)} = (2 - \alpha) \langle \xi \rangle, \quad (14)$$

where $\langle \xi \rangle$ is independent of α . We can easily turn this into

$$\frac{\langle x \rangle_\alpha}{\langle x \rangle_{\alpha=1}} = (2 - \alpha). \quad (15)$$

The sort of result that can be derived in the corresponding way for the six-quark cluster appears less useful. We envision one quark being struck and driven forward, and the residuum that remains “fragments” (or recombines) into a nucleon, that often goes backward, plus other stuff. The differential cross section is structurally

$$P(x, \alpha) = \frac{dN}{dx d\alpha} = \frac{g(x)}{2 - x} \cdot D(z = \frac{\alpha}{2 - x}). \quad (16)$$

The factor $g(x)$ is for the quark knockout and $D(z)$ is the fragmentation function. The first is a function of x since it involves distribution functions of quarks in the six-quark cluster, and the six-quark cluster is standing still in the lab. Argument z is defined earlier. The neatly derivable result is for average α at fixed x :

$$\langle \alpha \rangle_x = \frac{\int_0^{2-x} d\alpha \alpha P(x, \alpha)}{\int_0^{2-x} d\alpha P(x, \alpha)} = (2 - x) \langle z \rangle, \quad (17)$$

or,

$$\frac{\langle \alpha \rangle_x}{\langle \alpha \rangle_{x=1}} = (2 - x). \quad (18)$$

This result for the six-quark cluster contribution requires averaging over all α , and while the optimist may expect the six-quark contributions to dominate at high α , no-one expects them to do so at moderate α . So, this result seems untestable.

For $\langle x \rangle_\alpha$ details would have to be worked out for each special case. However, remembering the general result that $\langle x \rangle_\alpha$ decreases with increasing α and is zero kinematically when $\alpha = 2$ makes it likely that one will get something like the 2N result.

3.3 Possibilities at CEBAF

Can CEBAF with a 4 or 6 GeV beam be useful? We believe yes for the 6 GeV beam. The question centers around how much backward momentum is possible with an incoming electron of this energy, and how much data we can get in the scaling region.

3.3.1 Limits on backward proton momentum

The maximum directly backward proton momentum when an electron scatters from a deuteron is $(3/4)m_N$, or 704 MeV, but this is for the case of infinite incoming energy. If the energy is finite, the magnitude of the maximum backward momentum is reduced. For example, with $E = 4$ GeV and $Q^2 = 1$ GeV, the maximum directly backward proton momentum is

$$p^z \leq .600 m_N = 564 \text{ MeV}, \quad (19)$$

which is still a decent backward momentum. (For a 6 GeV electron beam and the same Q^2 , the maximum p^z is 609 MeV.)

These results were obtained with the help of

$$\alpha m_N \leq \frac{1}{2} \left(m_d + \nu - \sqrt{\nu^2 + Q^2} \right) \left\{ 1 + \sqrt{1 - \frac{4m_N^2}{2m_d\nu - Q^2 + m_d^2}} \right\}, \quad (20)$$

and also, for no transverse momentum,

$$p^z = m_N \frac{\alpha^2 - 1}{2\alpha}. \quad (21)$$

For $\nu \rightarrow \infty$, we recover the limits cited, for example, in [2]. Now at finite energy and for a given Q^2 we maximize the limit on p^z by maximizing ν . We have

$$\nu = E - \frac{Q^2}{4E \sin^2(\theta/2)} \leq E - \frac{Q^2}{4E}. \quad (22)$$

For $Q^2 = 1$ GeV² and $E = 4$ GeV, we get $\alpha \leq 1.77$ and p^z as quoted above.

Much of the limitation actually comes because fixing Q^2 for a given incoming energy puts a lower limit on Bjorken x . Since this is also the momentum fraction of the struck quark in the lab frame, it means that the struck quark is not moving forward as fast as possible, and the residuum is then not moving backwards as fast as possible. For the 4 GeV beam and the present Q^2 limit, $x_{min} = 0.14$. If we went to infinite energy, but maintained this value of x , α would still be limited by $\alpha \leq 2 - x_{min} = 1.86$.

3.3.2 The scaling window

Using the formulas with the scaling result for F_{2n} requires that we be in the scaling region. This will squeeze the allowed range of x to be narrower than the kinematic limits.

Scaling requires, at a minimum, that Q^2 be above 1 GeV² and that W (the photon plus single target nucleon c.m. energy) be above 2 GeV, out of the resonance region.

The lower limit on Q^2 sets a lower limit to x , since for a given energy there is a maximum energy transfer ν possible. The maximum ν comes for backward electron scattering and leads to $\nu \leq E - Q^2/4E$ or

$$x_{min} = \frac{4 E Q_{min}^2}{2m_N (4E^2 - Q_{min}^2)}. \quad (23)$$

The lower limit on W sets an upper limit on x . Applying the limit to the final state that comes from striking the neutron in the 2N correlation model gives

$$W^2 = (p_n + q)^2 = m_N^2 + 2m_N \nu (2 - \alpha) - Q^2 \geq W_{min}^2, \quad (24)$$

for a given α of the backward proton and letting $p_n^2 = m_N^2$. This leads to a limit, also reached for 180° scattering, that

$$x_{max} = \frac{2E (2m_N E(2 - \alpha) - (W_{min}^2 - m_N^2))}{m_N (4E^2 + (W_{min}^2 - m_N^2))}. \quad (25)$$

The curves giving x_{max} and x_{min} are shown in Fig. 3 for $\alpha = 1.4$. The scaling region is the region between the two curves.

For $\alpha = 1.4$, the scaling region includes only a short span of x for electron energy $E = 4$ GeV, but the span increases greatly for $E = 6$ or 8 GeV. The span for 8 GeV is over half of the maximum possible at any energy, and not too much less than could be got at 15 GeV. For increasing α , which corresponds to increasing velocity of the struck neutron away from the photon, larger energy in the laboratory is needed to reach the scaling region.

3.3.3 *Outside the scaling region*

Do we need to be in the scaling region? Our formulas for the 2N correlation are simplest to evaluate there, and we don't know how to guess at forms for production off the 6q component, so our comparison case is gone. But, from the 2N viewpoint, the purpose of scattering off the neutron is to free the proton—nothing more. If all we want is a yes/no on the 2N correlation model, we could take a measured cross section for producing backward protons and divide by a cross section for scattering off a neutron at the correct values of the incoming variables, and see if the result depends only upon α and p_T .

The disadvantage of doing this may be more practical. Driving the struck particles forward forcefully separates them greatly in momentum space from the backward proton and reduces the final state interactions, which we have neglected in our discussion. As we leave the scaling region, it can mean that the energy transferred to the forward moving particles is low enough that we need to rethink our attitude towards final state interactions.

3.4 *Possibilities with muons at higher energy*

At Fermilab, for example, experiment E665 scattered 490 GeV muons from deuterium and xenon targets in a streamer chamber so as to be able to observe backward charges. A later version of experiment E665 has capability to observe backward neutrons, and

uses carbon, calcium, and lead as its heavier targets. The x dependence of the ratio R_1 remains a good observable. There is a clear advantage in having higher energy as one does not have to think about the “scaling window.” It is virtually the full possible kinematic range.

Also there is some rate advantage in measuring backward neutrons instead of protons. For the 6q model, one can work out that if the 6q valence configuration dominates, as it should at high backward nucleon momenta, then the ratio of neutron rate to proton rate should be 3/2 and be independent of x . (One uses a combinatoric argument to get ratios of proton and neutron production from u^3d^2 and u^2d^3 residua, and some discussion of this appears in [3].) The result for the 2N model is different, since one has

$$\frac{\sigma_{2N}(\mu d \rightarrow \mu n X)}{\sigma_{2N}(\mu d \rightarrow \mu p X)} = \frac{F_{2p}(\xi)}{F_{2n}(\xi)}, \quad (26)$$

so that the ratio should vary from about 1 at low ξ to about 4 at high ξ . Thus backward neutrons are always produced more copiously, and this is an interesting additional observable if one has the capability of detecting both flavors of nucleons.

We might also note that study of backward nucleons from a deuteron target is a study of the “target fragmentation region” and is best and most easily carried out in the rest frame of the target, i.e., the lab. Data presented in the photon-target c.m. is not equally useful.

4 Conclusion

Study of the deuteron should be pursued since deuterons are our chief source of information about neutrons and we should understand this source. Further, the behavior of the deuteron state at short range gives information about the short range dynamics of strong interaction QCD.

We have suggested a measurement to learn what the short range wave function of

the deuteron is. Namely, examine the shape of the measured differential cross section for electroproduction of backward protons or neutrons from a deuteron target, and take its ratio to what would be expected for deep inelastic scattering from a free neutron or proton, respectively. A high momentum backward nucleon acts as a tag isolating events where the initial material in the deuteron was tightly bunched. The x dependence or lack of x dependence of the ratio is a signal that is distinct for the extreme cases of a pure 2N or pure 6q cluster. We have presented simple model estimates of the size of effects that may be seen, showing that factors of two differences from maximum to minimum may be expected in the 6q case, whereas no maximum to minimum difference is expected in the 2N case.

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A Appendix: Distributions for six-quark clusters

We apply the notation $|6q\rangle$ or $6q$ to label the situation when the neutron and proton are melded and lose their individual identity. This notation emphasizes the fact that standard QCD quark parton model considerations should be applied to this interesting multiquark object. Therefore, for a generic $|nq\rangle$ state the sea, valence, and gluon distribution functions (times z) are written

$$\begin{aligned}\overline{U}_n(z) &= A_n(1-z)^{a_n} \\ V_n(z) &= B_n z^{1/2}(1-z)^{b_n} \\ G_n(z) &= C_n(1-z)^{c_n},\end{aligned}\tag{27}$$

where z is the fraction of the total cluster momentum. The coefficients and powers are determined in [9] by appealing to standard normalization and momentum conservation considerations along with input information from experimental study of the $n = 2$ (pion) and $n = 3$ (nucleon) situations. As a result, 3 cases were developed to illustrate the sensitivity to small changes in the power of $(1-z)$, i.e., $(a_6, b_6) = (11, 9)$ for case A; $(11, 10)$ for case B; and $(13, 10)$ for case C.

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Figure 1: A putative ratio R_1 assuming the measured cross section for backward protons in deep inelastic scattering is dominated by 6-quark configurations and using the LS model [9] for the distribution functions of the 6-quark cluster. In general, R_1 is the measured cross section (sans some kinematic factors) divided by the neutron structure function F_{2n} . If backward proton production were dominated by 2-nucleon correlations, the plot would be flat. The plot is for fixed $\alpha = 1.4$ (momentum 322 MeV for directly backward protons) and uses CH [10] distribution functions to obtain F_{2n} . The heavy curve is from LS parametrization A, the normal curve is from parametrization B, and the dotted curve is from parametrization C. The vertical units are arbitrary as the fragmentation function $D_{p/5q}$ is not normalized.

Figure 2: Like Fig. 1 except that F_{2n} is gotten from the CTEQ [11] distribution functions, specifically from the set CTEQ1L for $Q^2 = 4 \text{ GeV}^2$. The heavy curve is from LS parametrization A, the normal curve is from parametrization B, and the dotted curve is from parametrization C. The curves turn up as $x \rightarrow 0.6$ ($\xi \rightarrow 1$) because the CTEQ distribution functions all approach zero at the upper limit faster than $(1 - \xi)^3$, and the fragmentation function we use goes to zero as $(1 - z)^3$.

Figure 3: The scaling window for $\alpha = 1.4$. Values of x between the two curves can be reached in the scaling region for a given incoming electron energy E , given in GeV above. The lower curve is set by the requirement that $Q^2 \geq 1 \text{ GeV}^2$ and the upper curve by the requirement that $W \geq 2 \text{ GeV}$. The curves begin at $E = 3.72 \text{ GeV}$ for this α .

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